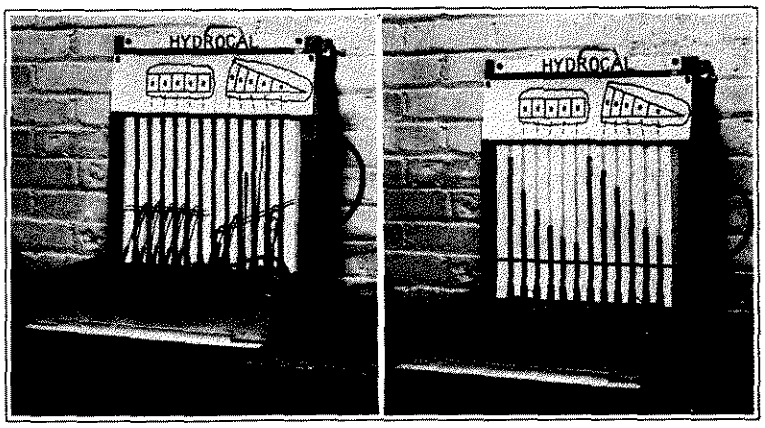
THE HYDROCAL

A Hydrodynamic Calculating Machine for Solving Unsteady-State Problems in Heat Transfer and Other Types of Diffusion[1]

A. D. MOORE
University of Michigan, Ann Arbor, Mich.



A. Original position *B.* In operation

FIGURE 1. DEMONSTRATOR HYDROCAL

IN HEAT transfer we must deal with heat flow, thermal resistance, temperature differences causing flow, and thermal storage. When these physical elements are combined in such a way as to present a problem in the transient state, the solution of the problem, in all but a few cases, presents typical difficulties of great magnitude. The Hydrocal is a machine of hydrodynamic type; its mission is to imitate, hydrodynamically, a transient heat problem and to solve it. If it can do this, obviously it will apply, in general, to any field of endeavor in which are involved potential difference, flow of material, resistance to flow, and capacity to store the material flowing. Chemical diffusion and underground water supply are examples of fields of application other than heat.

Until a short time ago the only Hydrocal in existence was a simple, small demonstrator (Figure 1). It is most easily explained in terms of one of the simplest heat transient cases—the heating of a rod from one end, no heat flow being permitted from the other end or from the side area. The rod is pictured in the left-hand drawing at the top of the instrument (Figure 1*A*). It will be heated from the left end, all other rod surfaces being covered by an ideal insulation that will permit no heat flow. Other than the exact mathematical solution, any solution for the temperatures achieved from time to time at any position in the rod must be approximate. The approximation consists in dividing the rod into the five incremental blocks shown and in arbitrarily concentrating the thermal capacity of an increment at its center. If a more exact solution is desired, more increments can be used. After such a division is made, a solution can be achieved by a laborious step-by-step method (possibly assisted by graphical tricks) or it can be solved on a Hydrocal.

On the face of the apparatus are mounted twelve glass tubes, called "standpipes." In the set-up shown in Figure 1, the six to the left were assigned to the rod solution. The extreme left standpipe has a special function, to be described. The other five, located immediately under the drawing of the rod, are assigned to represent the five thermal capacities of the five increments. Water in the standpipe represents stored heat in the rod increment. Standpipe capacity to hold water, per inch rise, represents the increment's heat capacity.

Thermal resistance, or resistance to heat flow, now needs a proper representation in terms of water flow and resistance offered to it. Small-bore flow tubes are shown in Figure 1*A*, tilted up until they rest among the standpipes. The thermal resistance from the center of one increment to that of the next is represented by a constant resistance to water flow offered by the tubes. The resistance is constant because only streamline flow is permitted in the tubes. At the left-hand end of the rod there will be a thermal resistance due to one-half an increment, plus a surface heat-transfer resistance. This is represented by a properly selected flow tube at the left end of the set-up.

To work the problem, a high tank supply of water is necessary whose high level represents the temperature, for example, of a hot gas that will be used to heat the end of the rod. The tank is within the apparatus and is not visible in Figure 1*A*; but the tank is directly connected to the extreme left-hand standpipe. All that remains is to adjust the water levels in the five "increment" standpipes to an initial low level, corresponding to the initial low temperature of the rod. This is done, and the level used is that shown by the crossbar placed across the face (Figure 1*B*). Connections are opened, and in a few minutes water levels as shown by the five increment standpipes are achieved. These levels can be directly interpreted as temperatures of the centers of the various increments.

Solution of Problems

The accurate solution for this problem, beginning with the equation as derived by Carslaw,[2] covers several computing

[1] The name "Hydrocal" is coined from *hydro* (water) and *calor* (heat).

[2] "Mathematical Theory of the Conduction of Heat in Solids," New York, Macmillan Co., 1921.

sheets and takes a day or more. The Hydrocal solves the problem in a 5- or 10-minute run.

Figure 1 also shows a set-up for demonstrating the case of heating a cylinder, initially at a low temperature, by heating its round surface in some manner. The case shown corresponds to immersing a long cylinder in a hot liquid, conditions being such that the surface heat-transfer resistance was essentially zero. Any other surface condition can be simulated if need be. In Figure 1*A* the flow tubes for the cylinder case (resting among the six standpipes to the right) are increasingly long from left to right. This corresponds to the increasing thermal resistances found from the center of one concentric shell to the center of the next, as we go inward. Again, in the cylinder set-up the left-hand standpipe of the group of six is directly connected to the water tank, and the five remaining standpipes represent the five incremental shells of the cylinder. Since the standpipes in the demonstrator are of equal cross section or capacity, the cylinder was divided into five shells of equal volume. Flow tube resistances were then selected to correspond to resistances as found in the cylinder. Figure 1*B* shows the "temperatures" arrived at after a few minutes of running time. Initial levels, etc., were as in the rod case.

It was implied above that the thermal capacity of a cylindrical shell was arbitrarily moved to the center of the shell. This does not mean that its position was at the mean radius, but at the "thermal capacity center"—that position, radially, which would divide the total shell capacity in half.

As to the arbitrary placing of thermal capacity at given centers, or radii, etc., there is at least one other treatment that can be adopted in place of the centering method adopted above. That would be to move each half of an increment's capacity to the boundary of the increment, each moving to the nearer boundary. This method is to be preferred when surface temperatures need to be solved directly.

Large-Scale Hydrocal

Experience gained in building and using the small demonstrator was incorporated in the design of a large-scale Hydrocal. Construction of the large unit was initiated during the summer of 1935. One of the problems demonstrated for this symposium is shown in operation in Figure 2.

The unit has two adjustable supply tanks; only one appears in Figure 2 (at the right-hand side of the machine). The pulleys operate the tanks. Each tank is directly connected, through large rubber tubing, to one of the two standpipes at the extreme left. The remaining eighteen standpipes are available for representing as many as eighteen increments in a solid being studied.

A large pipe, acting as a common manifold (not visible in the photograph) runs across the machine. Any or all of the twenty standpipes can be isolated from or connected to this manifold by operating the switch mechanisms at the extreme lower front edge of the machine. For example (counting from the left), all the switches are shown closed except Nos. 2, 3, 9, and 15. Thus, one of the tanks, which is connected to standpipe 2, is now also connected to standpipes 3, 9, and 15. The levels in these standpipes so indicate.

The "resistance box" is the large box in the foreground. In it are the flow tubes, permanently mounted with their nipples standing vertical, ready to be connected in series with each other or to the standpipes. At the time Figure 2 was taken, the resistance box had mounted in it only four sets of flow tubes (eighteen tubes per set), the resistances being 1, 2, 3, and 4 units in value. Thus any combination of whole-number resistances could be added up by series connection, from 1 unit to 10 (or somewhat more) units of resistance in a path. Additional sets will soon be installed having values of 10, 20, 30, and 40 for resistance values. These flow (or resistance) tubes are calibrated within 0.5 per cent.

Immediately behind the large resistance box a shallower, narrower box or tank is seen; the rubber connectors terminate in it. In this tank are the standpipe manifolds. Each standpipe connects to a manifold with eight outlets or nipples. Normally, most of the eight nipples would not be needed and are blocked off. However, as many as six might be needed, since in a three-dimensional problem any one incremental volume in a solid would have six flow paths, from it to the six adjacent incremental volumes.

Back of and above the rubber connectors, but in front of the lower ends of the standpipes, are some horizontal bars. These bars terminate in the cam mechanism at the right. All this amounts to a set of gang shut-off mechanisms, the three gangs each accounting for a group of six standpipes. They operate by squeezing a rubber tube that connects a standpipe manifold to the standpipe. Thus, at any instant, the changing levels in the standpipes can be "frozen" for any one of several reasons. Operation is resumed at any later time desired. In order to describe another feature of this mechanism, suppose the cams are all thrown to the left initially, thereby closing off the standpipes from the flow system. The rubber tubes thus squeezed shut will, when the shut-off is thrown open to begin operation, spring to normal shape; the volume in the rubber tube in the squeezed region will increase, causing the levels in all standpipes to drop by about 0.1 inch. To eliminate this undesirable result, the cams are thrown to the right, whereupon a short "dummy" rubber tube (one for each position) is squeezed to give a compensating effect and put the standpipe level back to where it should have remained.

The "level-freezing" mechanism is highly desirable for several reasons: (*a*) It is in constant use during the setting-up part of the operation when flow tubes are being connected. (*b*) It may be used to stop the operation simply in order that the operator may take his time to read at suitable intervals. (*c*) It enables him to solve problems in which surface heat-transfer coefficients, for example, may change with the change in temperature drop

The Hydrocal is a calculating instrument operating on hydrodynamic principles. Transient problems in diffusion of heat, chemical diffusion, or other fields where diffusion laws prevail, can be set up on the Hydrocal and worked. Explanation can be made in terms of a transient heat situation in a simple solid—a slab being heated from one side. The slab is divided into a suitable number of incremental slices, and the Hydrocal is then set up to imitate all of the heat characteristics of the slab.

This paper is a progress report and accompanied the demonstration on December 30, 1935, at Yale University, of the first large-scale Hydrocal built. The demonstration marks the two-year point in the development of this new type of calculating machine.

at the surface. During a shutdown, the flow path representing surface thermal resistance can be reconnected to have a new value as dictated by surface temperature drop so far reached. (*d*) If the thermal conductivity of the solid materially changes with temperature, it may be desirable to break the complete run into several time increments and, during shutdown, make similar flow path changes in every flow path used. (*e*) If thermal capacity varies with temperature, shutdown between suitable time increments can permit changes in the capacity of the standpipes (by a method outlined later). (*f*) It becomes possible to simulate the production (or withdrawal) of heat within the solid itself, as in the case of heat liberated by chemical change of state or of heat produced in electrical apparatus in the solid itself. This is accomplished by using the time-increment method again; and during the shutdown, when the gang shut-offs have "frozen" the standpipe levels, the lower front valve switches are manipulated to introduce (or withdraw) predetermined amounts of water into each standpipe. Thus, we will already have computed the heat to be evolved in a given volume increment for the next time increment; it is a simple matter to compute further the rise in temperature this will give to that solid increment. We then introduce water in the standpipe to change the "temperature" from the "frozen" value to the computed value. The gang shut-off is then opened, and the Hydrocal is permitted to operate through the next time increment.

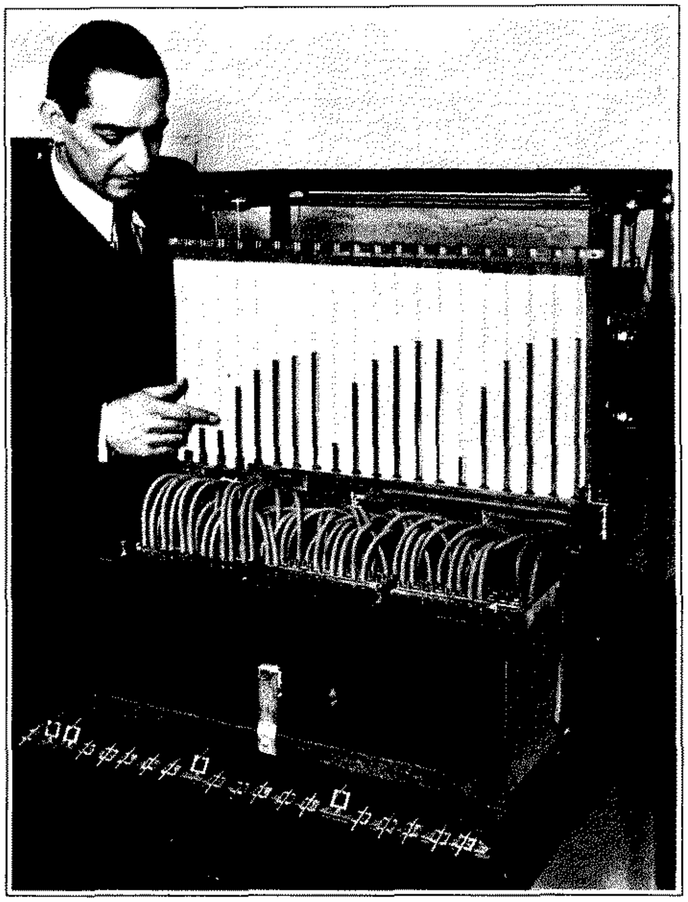


FIGURE 2. THE FIRST LARGE-SCALE HYDROCAL

The apparatus is in operation, solving for temperatures at fifteen different points within a solid cylinder; the cylinder is being cooled from both ends as well as from the round surface.

Included by implication in the preceding statement is the possibility of presetting the "temperatures," or standpipe levels, to any predetermined temperature distribution at the beginning of operation.

The matter of simulating various (or varying) thermal capacities was mentioned above. This will be done by a part of the apparatus (not yet constructed) called the "tilter." Each of the fixed standpipes (Figure 2) will be freely connected to a tiltable standpipe in the tilter. By selecting a proper standpipe in the tilter as to cross section, and by the additional device of tilting it to a proper angle, the particular standpipe position can be adjusted, within limits, to any desired capacity.

Figure 2 shows the Hydrocal in operation, when solving for temperatures at fifteen different points within either half of a cylinder which is being cooled, not only from the cylindrical surface but from the ends as well. The cylinder was divided radially into five shells of equal volume, as in Figures 1 and 2. It was further sliced by planes normal to the axis, into six equal slices. Since the case is symmetrical about the center plane, only half the case need be solved. In one-half there will be fifteen increments. The group of five standpipes near the author's hand (Figure 2) represents the temperatures at the thermal centers of the five concentric shells in the left-hand slice of the cylinder; the right-hand group of five stands for the shells in the slice next to the center plane; and the middle group of five, the shells of the slice between these two. The "temperatures" of levels for the end-slice standpipes have fallen farthest, the middle group less fast, and the right-hand group most slowly, reflecting the fact that the end cooling is having its effect.

Conversion

A question always raised at demonstrations is, how is a problem carried over to and set up on the Hydrocal, and how are the Hydrocal values, as read, converted into answers to the problem? Space does not permit a full description of the conversion, but the principles are easily set forth. We can arbitrarily decide that one cubic inch of water shall represent one unit of heat, and that one inch of water rise in a standpipe shall represent one degree rise in temperature. Then we may calibrate a particular flow tube, at one-inch head of water, and find that it conducts one cubic inch of water in unit time. It would then have one unit of thermal resistance. If a standpipe is chosen that stores up one cubic inch of water in a one-inch rise, it has unit thermal capacity. A temperature scale mounted behind the standpipes would mean one degree for every inch.

Since a given Hydrocal can have only a limited range of flow tubes and standpipes, it will be obvious that the usual problem cannot possibly be fitted directly to the values available. Here is where the scale effect, or principle of similarity, enters. It may be necessary to divide all of the problem's thermal resistances by a factor of 100 in order to fit the flow tube resistances available. Then the Hydrocal will simply work the problem one hundred times as fast as phenomena will occur in the actual situation. If all thermal capacities had to be divided by 10 before being set up on the apparatus, then because of this factor alone the Hydrocal will operate ten times as fast as the actual physical transient. The temperature scale can likewise be changed. A similar type of factor enters to take care of temperature-viscosity effect on the flow tubes. Using water as the material of flow, viscosity (and therefore resistance) changes about 5 per cent for each 2° C. of fluid temperature change. The flow tubes are calibrated at 20° C. Since they are immersed in a constant-temperature bath, we simply need to take the prevailing

temperature of the bath and correct the time scale by using a numerical factor taken from a proper chart.

Development

Many problems incidental to the development of a workable type of apparatus have arisen. One of the earliest worries had to do with the possibility of clogging that might develop in a flow tube with an inside diameter of about 0.05 inch. For many months these tubes were given hundreds of opportunities, in water that was far from clean, to clog. Actual clogging effects have happened with exceeding rarity. The worry is now dismissed.

The meniscus effect in standpipes gave trouble for some months. The meniscus artifically raises the level. When a meniscus breaks here and there, owing to fouling of the standpipe, errors are introduced. A technic was worked out whereby a set of standpipes can be used for days, if not weeks, without cleaning.

Rubber tubing is used to connect flow tubes in series and to connect to standpipe manifolds. Such tubing offers resistance that is unwanted. But since resistance of a round duct, in streamline flow, varies inversely with the fourth power of the diameter, theory and tests show that the rubber tubing used offers a negligible resistance, if the flow paths are of about 4 or more units of resistance each.

Securing standpipes of constant diameter and uniform bore was, and may still be, a problem. The German-made constant-bore tubing would be ideal, but the price is discouraging. At the present writing a shipment of Pyrex tubing is on its way from the Corning Glass Works. This shipment, nominally all of one size, is held to specified limits through selections made by D. J. Carr, one of the writer's former students now employed by Corning. If this batch turns out to be satisfactory, there should be no difficulty in securing other diameters by the selection method.[3]

Manifolds and other parts are made of brass, with soldered joints. There is some evidence that corrosion products within the manifolds may form and lead to fouling of the system. This worry is being put on the shelf for the time being. Experience with the large Hydrocal should give a more definite answer. Brass nipples on manifolds and brass nipples on the ends of flow tubes permit the rubber connectors to make water-tight connections. But brass ages rubber and causes fouling and sticking. Electrotinning these parts seemed to be an ideal solution, since tin is widely used for containing distilled water, and tin does not attack rubber. Tinning received a setback recently when still another problem had to be solved: Mold would form in the water, and the cure was to add copper sulfate. The copper sulfate stripped off the tin and caused masses of "floaters" of a tin salt. This effect was so pronounced that operations had to be stopped while a centrifugal pump and filter combination was built to enable the water in the tank of a tube-calibrating apparatus to be cleared rapidly. These troubles reflect the fact that the writer's training in chemistry took place about twenty-three years ago. The same troubles led to his appreciation of the interest and helpfulness of a number of colleagues in chemistry, chemical engineering, and pharmacology, whose suggestions at times have proved invaluable. Unless a better suggestion is offered, the immersed parts of the large Hydrocal may have to be gold-plated. The eventual material for some of the parts will, no doubt, be one of the thermosetting plastics.

Flow Tube Research and Calibration

An interesting phase of the development was the problem of flow tube design. A strictly constant resistance can be achieved only when the tube is very long (in terms of diameters) and is kept straight. It is not feasible to make all of the tubes in the "resistance box" very long; this would unduly lengthen the time needed for making solutions. The shortest tube, called the "unit" tube, is only about 4 inches long. End effects alone cause a rapid rise in resistance for this tube for heads over one inch of water. Moreover, the tubes must be bent in some way to make a feasible design that permits of ready interconnection and that will not call for unduly long pieces of rubber connectors. Bends have little or no effect at low heads, but they increase resistance at higher heads at a rapid rate. Is it better, then, to curve the whole length of a 40-inch tube to a gentle radius, or should it be bent like a hairpin, with long straight parts and short (but sharply bent) curved parts? Theory, and the present state of hydrodynamics, do not yield a certain answer. In order to measure the characteristics of these flow tubes, a static-level calibrator was developed. With this calibrator, in tedious fashion, direct measurements of heads and flow rates were made on a number of tube designs. Work with this apparatus has brought out what definitely appears to be an anomalous variation of resistance with head increase for streamline flow in hairpin-style tubes. The anomaly has appeared so uniformly in various styles of tubes that a further investigation may be made and the findings reported elsewhere.

As soon as some standard tubes were created by the slow, accurate methods available in the static-level calibrator, a new "dynamic" type calibrator was developed. The new device, recently completed, compares a standard tube against the calibrated tube. It is possible to produce, from raw stock, about three calibrated tubes per hour; calibrations (in terms of standard) are in error by less than 0.5 per cent.

The longest tubes, with 40 units of resistance, will be about 160 inches long and will have a virtually constant resistance over the entire 10-inch head used in the Hydrocal. The shortest, or 1-unit tube, will be constant over a 1-inch head or less. However, if a unit tube is ever used to connect directly between two standpipes, the fact that its resistance is low will guarantee that the difference in head acting on it will be suitably low.

Standpipe Design

Standpipe design offers an interesting set of limitations. A small-bore standpipe would offer a resistance, when water level is changing, that would be entirely unwanted; such a resistance has no counterpart in the physical problem being represented. Also, too small a bore would magnify meniscus effects. Standpipes of very large cross section would minimize meniscus and fouling troubles, but would undesirably slow up the apparatus. The design adopted strikes between the two difficulties and practically eliminates both. It would be very advantageous if much taller standpipes could be used. The result would be increased accuracy in reading levels and a decrease in meniscus effects. Two conditions operate to prevent much increase over something like a 10-inch height: (*a*) It would increase the resistance to flow offered by the pipe itself. (*b*) Such high heads would carry the flow tubes beyond the range within which their resistance is approximately constant.

Accuracy

It is expected that, under usual conditions, flow tube paths will have resistances giving an error of less than 2 per cent, and often of less than 1 per cent. The standpipes should have cross sections that are within 2 per cent, or less, of a nominal value. Thus, the largest error that the apparatus collectively could produce would usually be under 3 per cent. No general statement can be made on errors aside from these.

[3] Of twenty tubes so selected and supposed to be within 1 per cent of the desired section, sixteen were within the limits, and 4 were within 2 per cent.

Dividing the original physical case into pieces or increments will introduce errors, the magnitude depending on the degree of subdivision. Other errors will come in when the run must be broken into time increments. Still others will come from inaccuracies present in heat transfer and other constants. However, assuming constants to be known (as must be assumed no matter how a problem is solved) and assuming that enough increments are used largely to eliminate errors from this type of approximation, the apparatus should be made to yield very satisfactory results in a large variety of applications.

A single large Hydrocal, with but eighteen usable standpipe positions, would apply only to the simpler types of three-dimensional problems. However, the design can be such that Hydrocal units may be placed side by side and interconnected without limit.

Present plans call for gaining some experience in operating the large Hydrocal before attempting to redesign it. It is practically a hand-built job, and some changes will have to be made before production by some instrument maker can be arranged.

Received February 11, 1936.

◆ ◆ ◆

Temperature in INDUSTRIAL FURNACES

Interpretation and Use to Measure Radiant Heat Flux

H. C. HOTTEL, F. W. MEYER, AND I. STEWART
Massachusetts Institute of Technology, Cambridge, Mass.

The significance of temperature measurements in industrial furnaces is discussed. The true gas temperature has less utility, except in making heat balances, than the uncorrected reading ordinarily obtained with a protected couple; the latter measures the rate at which heat would be transferred if the couple were replaced by a surface at the temperature of the heat sink. An instrument consisting in principle of a pair of oriented thermocouples is shown to be capable of measuring the actual rate of heat flow across any plane in a furnace. The instrument was tested at rates of radiant heat flow across a plane of from 5000 to 18,000 B. t. u. per square foot per hour, with an average error of 4 per cent in eight tests. Its application to a study of uniformity of heat distribution in furnaces is discussed.

COMMON to most developments in the design of processing equipment is the transition from a period of satisfaction with the prediction of over-all performance to one of profit from a consideration of the detailed performance of the various elements of the structure. In the field of high-temperature heat transmission the empirical method of building a furnace like one which was found satisfactory has to some extent been replaced by designs influenced by calculations based on an analysis of the mechanism of the heat transmission. Such calculations have, in general, had an objective no more ambitious than the prediction of over-all performance, with no consideration of how the heat is distributed to the various heat-receiving surfaces in the radiant section of the furnace. In the case of furnaces with heat-receiving surfaces at a relatively low temperature, knowledge of such heat distribution was not essential. The high-pressure processes of chemical industry, however, involve the transmission of heat through surfaces at elevated temperatures at rates which must not exceed definite values imposed by considerations of strength of the materials of construction. Hence the spread between average and maximum heat-input rates per unit of surface determines the amount of surface required; and, since the unit cost of such surfaces is high, the need for equalization of heat flow in the various parts of such furnaces becomes evident. The first step in that direction is the development of instrumentation for measurement of heat flow rates in the various parts of furnaces; that is the objective of the work here described.

A few sporadic attempts in this direction are recorded in the technical literature. The instrument used was a "thermaprobe," a metal body whose rate of heat absorption can be measured by observing its rate of rise of temperature. The earliest device was a spherical probe (*1*). Later a flat plate, protected below and on the sides, was used to determine the heat transfer in a billet-reheating furnace, the instrument being held above and close to the stock (*6*). Just this year a boiler furnace was investigated with a short cylindrical probe (*5*) of the same diameter as a boiler tube; the instrument was held at various points along a tube axis so that the convection and radiation characteristics should correspond exactly to those of a tube. (Presumably the tubes in the lowest banks were double-spaced and the instrument was inserted in the blank rows.) Finally the A. S. M. E. Committee on Absorption of